Law of Sines

Law of Sines - Used when given AAS or ASA

- Triangles will have ONE solution

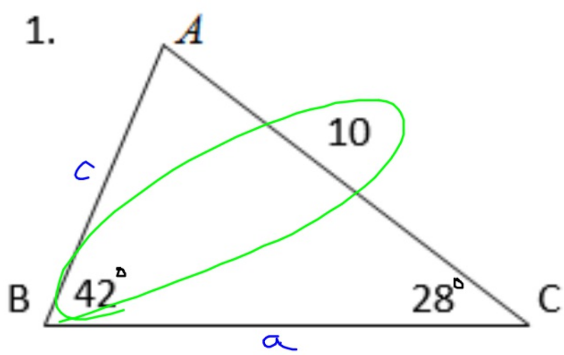
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example: AAS

1.



$\underline{AAS}$ - L. of S.

$$\frac{\sin 42}{10} = \frac{\sin 28}{c}$$

$$\frac{c \cdot \sin 42}{\sin 42} = \frac{10 \cdot \sin 28}{\sin 42}$$

$$\boxed{c = 7.02}$$

$$\boxed{\begin{aligned} m\angle A &= 110^\circ \\ c &= 7.02 \\ a &= 14.04 \end{aligned}}$$

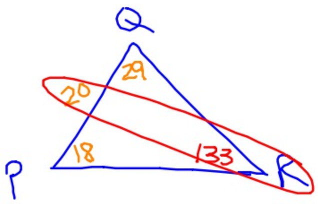
$$\frac{\sin 42}{10} = \frac{\sin 110}{a}$$

$$\frac{a \cdot \sin 42}{\sin 42} = \frac{10 \cdot \sin 110}{\sin 42}$$

$$\boxed{a = 14.04}$$

Example: ASA

2. $\Delta PQR,\ \angle Q = 29^\circ, \angle P = 18^\circ$, r =20



ASA : ∠ . of S.

$$\frac{\sin 133}{20} = \frac{\sin 18}{p}$$

$$\frac{p \cdot \sin 133}{\sin 133} = \frac{20 \sin 18}{\sin 133}$$

$$p = 8.45$$

$$\frac{\sin 133}{20} = \frac{\sin 29}{q}$$

$$\frac{q \cdot \sin 133}{\sin 133} = \frac{20 \cdot \sin 29}{\sin 133}$$

$$q = 13.26$$

$$\boxed{\begin{array}{l} m\angle R = 133^\circ \\ p = 8.45 \\ q = 13.26 \end{array}}$$

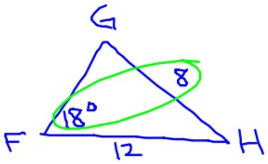
Law of Sines - Also used when given SSA

- This is a special case called the "Ambiguous Case"
- Triangles will have ONE solution, TWO solutions or NO solutions

What is the Ambiguous Case?

Example 1: Two Solutions

In ▲FGH, m<F=18°, f=8, and g=12



SSA: L. of S.
(Amb. Case)

1.

$$\frac{\sin 18}{8} = \frac{\sin G}{12}$$

$$\frac{8 \cdot \sin G}{8} = \frac{12 \cdot \sin 18}{8}$$

$$\sin G = \frac{12 \sin 18}{8}$$

$$\sin^{-1}\left(\frac{12 \sin 18}{8}\right) = G$$

$$\boxed{m\angle G = 27.61°}$$

2.

$m\angle H = 180 - 18 - 27.61$

$m\angle H = 134.39$

3.

$$\frac{\sin 18}{8} = \frac{\sin 134.39}{h}$$

$$\frac{h \cdot \sin 18}{\sin 18} = \frac{8 \sin 134.39}{\sin 18}$$

$h = 18.50$

1st Δ

$\boxed{\begin{array}{l} m\angle G = 27.61° \\ m\angle H = 134.39° \\ h = 18.50 \end{array}}$

2nd Δ

$\boxed{\begin{array}{l} m\angle G = 152.39° \\ m\angle H = 9.61° \\ h = 4.32 \end{array}}$

Check for 2nd Δ

1. Subtract 1st ∠ you found from 180

$180 - 27.61 = \boxed{152.39}$ potential new ∠G

2. Add potential new ∠ to angle you were given.

- If < 180 : yes 2nd Δ
- If > 180 : no 2nd Δ

$152.39 + 18 = 170.39$

< 180, yes 2nd Δ

2nd Δ

$\boxed{m\angle G = 152.39}$

$m\angle H = 180 - 152.39 - 18$

$\boxed{m\angle H = 9.61°}$

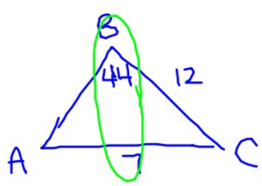
$$\frac{\sin 18}{8} = \frac{\sin 9.61}{h}$$

$$\frac{h \sin 18}{\sin 18} = \frac{8 \sin 9.61}{\sin 18}$$

$h = 4.32$

Example 2: No Solution

In ▲ABC, m<B=44°, a=12, and b=7.



SSA: L. of S.
Amb Case

$$\frac{\sin 44}{7} = \frac{\sin A}{12}$$

$$\frac{7 \cdot \sin A}{7} = \frac{12 \sin 44}{7}$$

$$\sin A = \frac{12 \sin 44}{7}$$

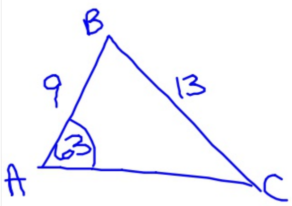
$$\sin^{-1}\left(\frac{12 \sin 44}{7}\right) = A$$

$\downarrow$
error

No $\triangle$

Example 2: One Solution

In ▲ABC, m<A=63°, a=13, and c=9.



SSA: L. of S
(Amb Case)

$$\frac{\sin 63}{13} = \frac{\sin C}{9}$$

$$\frac{13 \sin C}{13} = \frac{9 \sin 63}{13}$$

$$\sin C = \frac{9 \sin 63}{13}$$

$$\sin^{-1}\left(\frac{9 \sin 63}{13}\right) = C$$

$$\boxed{m\angle C = 38.09^\circ}$$

Check for 2nd Δ.

$180 - 38.09 = 141.91$

$141.91 + 63 = 204.91$

No 2nd Δ.

$m\angle B = 180 - 63 - 38.09$

$$\boxed{m\angle B = 78.91^\circ}$$

$$\frac{\sin 63}{13} = \frac{\sin 78.91}{b}$$

$$\frac{b \sin 63}{\sin 63} = \frac{13 \sin 78.91}{\sin 63}$$

$$\boxed{b = 14.32}$$

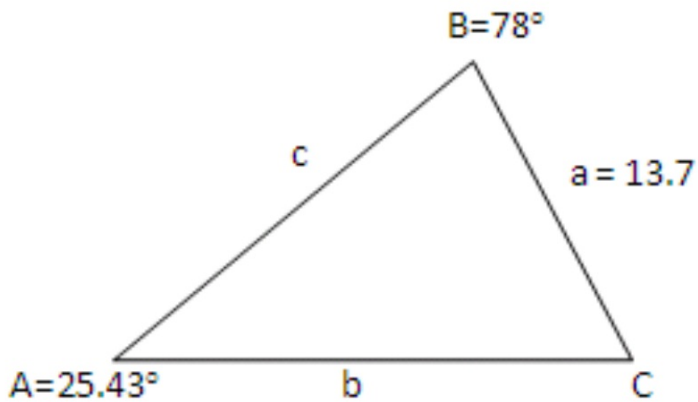
one Δ

$$\boxed{\begin{array}{l} m\angle C = 38.09^\circ \\ m\angle B = 78.91^\circ \\ b = 14.32 \end{array}}$$

Practice: Pg. 803-804
#24-26, 27, 29, 33

Warm Up: Solve for all the missing sides and angles

1. $a = 13.7$, $A = 25.433°$, $B = 78°$



Law of Cosines- Used when given SSS or SAS.

$$a^2 = b^2 + c^2 - 2\mathrm{bc}\left(\cos\mathrm{A}\right)$$
$$b^2 = a^2 + c^2 - 2\mathrm{ac}\left(\cos\mathrm{B}\right)$$
$$c^2 = a^2 + b^2 - 2\mathrm{ab}\left(\cos\mathrm{C}\right)$$

Example:

In ▲ABC $a=8$, $b=19$, and $c=14$. Determine the values of all the missing angles.